

DYNAMIC ADJUSTMENTS OF THE OPTIMAL HEDGE RATIO

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ABSTRACT

With the changing nature of the real world it is unrealistic to assume that the hedge ratio estimated from historical data over a long period is constant. The dynamic nature of the hedge ratio presents a practical problem in performing OLS to compute the optimal hedge ratio. Five econometric models for handling the nonstationary optimal hedge ratio are analyzed. They are: the pure random coefficient model, the Bayesian model, the univariate ARCH model, the bivariate ARCH model, and the generalized changing parameter model. In addition, these methods are applied to examine the dynamic hedging potentials for stock portfolios. It is found that the return-risk relatives from the dynamic strategies are substantially higher than those of the static hedging strategies while the return distributions of the former are also more positively skewed than the latter. The findings demonstrate the superiority of the more sophisticated dynamic hedging strategies.

I. INTRODUCTION

One of the important roles of futures markets is to render an efficient facility for hedging. In order to perform an effective hedge, it is critical to determine the optimal hedge ratio. Among the hedging models, Johnson's (1960) minimum-variance hedge ratio is the most popular one employed in empirical studies (see Ederington, 1979; Frankle, 1980; Hill and Schness-weis, 1981, for eg.). Previous studies have suggested that the optimal hedge ratio (h) be estimated from the slope coefficient of the following linear regression:

$$R_{st} = a_0 + hR_{ft} + \epsilon_t \quad (1)$$

where R_s and R_f denote the returns to the cash position and the futures position, respectively; and ϵ symbolizes the random error. Hedging effectiveness can be determined by examining the R^2 value of the OLS regression.

For the h estimate to be of practical use, the stability of h is generally assumed. This is especially true when the agent attempts to roll over the futures contract during the hedge period. However, the instability of the hedge ratio may stem from several sources. First, theoretically, Samuelson (1965) postulated that if futures prices follow a mean reverting process, then the volatility of futures prices may increase as maturity approaches. Second, the joint probability distribution of returns to cash and futures positions may not be stable over time. Third, changes in hedge ratio may be attributed to some systematic components not included in Eq. (1) and/or to any random shock. For any of these reasons, the dynamic nature of the hedge ratio exhibits a practical problem in estimating an optimal hedge ratio from Eq. (1).

Accordingly, to address the issue of dynamic hedging strategy, we are to present and test five alternative methods to determine the dynamic hedge ratios in section II. Following that is a section which evaluates the relative

effectiveness of dynamic hedging strategies and static hedges. The comparisons are based upon several broader measures than the traditional one, the R^2 . Finally, a summary and several conclusions will be drawn.

II. METHODS AND RESULTS

In this constantly changing world, it is unrealistic to assume that the hedge ratio estimated from historical data over a long period is constant and applicable to the future. Five methods for handling the nonstationary optimal hedge ratio are analyzed. They are: the pure random coefficient model, the Bayesian model, the univariate ARCH model, the bivariate ARCH model, and the generalized changing parameter model. The pure random coefficient model is employed to correct the random variation of the hedge ratio estimate. The Bayesian and mixed estimator developed by Chen and Lee (1983) presents an explicit time-varying hedge ratio formula which can be utilized to estimate the hedge ratio at any point of time. The univariate ARCH model allows the underlying conditional variance to change over time and the variation is predicted by past forecast errors. This method not only provides a more efficient estimator than the OLS estimator, but also take into account the effect of possibly omitted variables from the estimated equation. The Bivariate ARCH model can be applied to construct a conditional covariance matrix for the unexpected returns in the cash market and futures market. The time-varying hedge ratios are then generated from the conditional covariance matrix. The generalized changing parameter model allows the hedge ratio to vary with factors not incorporated in the estimating equation as well as random shock.

Moreover, to examine the relative effectiveness of more sophisticated hedging strategies, empirical tests are carried out. Tests will utilize weekly returns data on the S&P 500 index and index futures gathered from April 1982 through December 1985. However, when examining the generalized

changing parameter model, we will employ options on individual stocks, options on the S&P 100 index, and options on the S&P 500 index futures as hedging instruments.¹

[1]. *Application of The Pure Random Coefficient Model*

Following Theil (1971), the hedge ratio can be allowed to fluctuate over time. The risk-minimization hedging equation can then be expressed as

$$R_{st} = a_0 + h_t R_{ft} + \epsilon_t \quad (2)$$

where the hedge ratio in period t is determined by

$$h_t = h_0 + u_t$$

Thus Eq. (2) can be written as

$$R_{st} = a_0 + h_0 R_{ft} + w_t \quad (3)$$

where

$$w_t = u_t R_{ft} + \epsilon_t,$$

and we assume

$$E(h_t) = h_0, \text{Cov}(\epsilon_t, u_t) = \text{Cov}(\epsilon_t, h_t) = \text{Cov}(\epsilon_t, \epsilon_{t-1}) = 0$$

The estimate of the variance of the hedge ratio, σ_u^2 , can be used to test for the randomness of the hedge ratio. If σ_u^2 is significantly different from

¹ The futures contracts utilized are the nearby contracts, while the options contracts used are the near-the-money options with time to expiration of less than 90 days.

zero, it may indicate that the hedge ratio varies randomly over time. Hence the ordinary least squares (OLS) estimate h is inefficient or just suboptimal.

In addition, the estimates of the pure variance, σ_e^2 , and the variance of the time-varying hedge ratio, σ_u^2 , can be obtained by performing a regression on the model

$$w_t^2 = b_0 P_t + b_1 Q_t + V_t \quad (4)$$

where

$$E(V_t) = 0, \quad P_t = 1 - \frac{f_t^2}{\sum f_t^2},$$

$$Q_t = f_t^2 \left[1 - \frac{2f_t^2}{\sum f_t^2} + \frac{\sum f_t^4}{(\sum f_t^2)^2} \right]$$

and

$$f_t = R_{ft} - \bar{R}_f$$

The estimates of the coefficients b_0 and b_1 are σ_e^2 and σ_u^2 , respectively.

Because of the heteroscedastic nature of the error term V_t , a generalized least squares (GLS) method is used to obtain efficient estimates of σ_e^2 and σ_u^2 . Also, the high multicollinearity between P_t and Q_t in Eq. (4) requires another adjustment. Theil and Mennes (1959) alleviated the problem by, in effect, substituting the value 1.0 and f_t^2 for P_t and Q_t , respectively. Thus an OLS regression can be performed on

$$w_t^2 = b_0 + b_1 f_t^2 + e_t \quad (5)$$

to test for the randomness of the h coefficient. Notice that the R^2 from the GLS can no longer assess the percentage reduction in the variability of

the cash position. A revised measure for this purpose will be suggested in the following section.

Table 1 presents the estimates of σ_e^2 and σ_u^2 for the stock index futures hedge. The OLS estimates of σ_u^2 are significant for the one-week and two-week hedges but not for the four-week hedge. It can be concluded that the use of the random coefficient hedge model is appropriate for the short hedging horizon in this sample.

Table 1
Results of Pure Random Coefficient Model
(Equation (5) and h_{GLS})

	b_0	b_1	h_{GLS}
SP1W	0.2044×10^{-4} (5.7376)	0.0070 (2.0062)**	0.9120 (50.0926)***
SP2W	0.3186×10^{-4} (3.4395)	0.0081 (2.4023)***	0.9236 (35.5113)***
SP4W	0.6578×10^{-4} (2.7202)	0.0009 (0.1879)	0.8994 (32.6244)***

h_{GLS} = GLS estimate of hedge ratio

(numbers in parentheses are t-statistics)

** . significant at 5% level

*** significant at 1% level

[III]. Application of The Bayesian Model

A Bayesian and mixed estimator of the time-varying systematic risk was developed by Chen and LEE (1983) from the pure random coefficient model Eq. (2). Employing this approach, we can represent the dynamic adjustment of the hedge ratio as

$$\hat{h}_t^* = \frac{\hat{h}_0 + \hat{\Gamma} x_t y_t}{1 + \Gamma x_t^2} \quad (6)$$

where

$$x_t = R_{ft} - \bar{R}_f$$

$$y_t = R_{st} - \bar{R}_s$$

$$\hat{\Gamma} = \hat{\sigma}_u^2 / \hat{\sigma}_\epsilon^2$$

The prior (\hat{h}_0) and the variance ratio ($\hat{\Gamma}$) can be estimated using the maximum likelihood method.

To show the usefulness of Eq. (6) in the hedging problem, Bayesian dynamic adjustment hedge ratios are estimated for hedging the stock portfolios with one-week, two-week, and four-week adjustment horizons. The results are presented in Table 2. Table 3 lists the results of the initial of OLS regression of Eq. (1). A closer inspection of Tables 2 and 3 reveals that the mean of the time-varying hedge ratios may not substantially differ from the OLS estimates. However, the pure error variances ($\hat{\sigma}_\epsilon^2$) in Table 2 are much lower than the OLS estimates of error variance presented in Table 3. The difference indicates that the OLS residual variance is overestimated.

Table 2
Bayesian Estimation Results

	Mean of h_t	Pure Residual Variance
SP1W	0.9225	0.1598×10^{-4}
SP2W	0.9285	0.2983×10^{-4}
SP4W	0.8964	0.6382×10^{-4}

h_t = Bayesian time-varying hedge ratio

Table 3
The Initial Ordinary Least Squares (OLS) Regression Results

	h^*	R^2	σ^2
SP1W	0.9026	0.9430	0.2408×10^{-4}
SP2W	0.9300	0.9557	0.4082×10^{-4}
SP4W	0.9039	0.9645	0.7074×10^{-4}

$$h^* = \text{the minimum-variance hedge ratio} = \frac{\sigma_{sf}}{\sigma_f^2},$$

R^2 = the coefficient of determination of the regression,

σ^2 = the residual variance,

SP1W = the one-week hedge using S&P500 index futures,

SP2W = the two-week hedge using S&P500 index futures,

SP4W = the four-week hedge using S&P500 index futures.

[III]. Application of the Univariate ARCH Model

Conventional studies in hedging analysis utilize historical data to estimate the simple regression Eq. (1). An assumption employed in Eq. (1) is that the error term is homoscedastic. If this assumption is violated, the estimates using OLS will be inefficient and the error variance estimate will be biased. Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) model which allows the conditional variance to change over time. The ARCH process is characterized by zero mean, serially uncorrelated process with nonconstant variance conditional on the past but constant unconditional variance. Applying this model, the hedge ratio estimation problem can be formulated as

$$R_{st} = a_0 + hR_{ft} + \epsilon_t$$

$$V_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_p \epsilon_{t-p}^2$$

$$\epsilon_t = R_{st} - (a_0 + hR_{ft})$$

where V_t represents the conditional variance.

The h and α parameters can be estimated by the maximum likelihood method. Moreover, to prevent the possibility of a negative conditional variance, all the α parameters are restricted to be non-negative and the sum of the α parameters is assumed to be less than unity [see Engle (1983)].

The significance of a p th order ARCH process can be tested via the following regression:

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \dots + \alpha_p u_{t-p}^2 + e_t \quad (7)$$

where u 's are the OLS residuals.

The null hypothesis that

$$\alpha_1 = \dots = \alpha_p = 0$$

will be rejected if the product of the sample size T and R^2 of regression Eq. (7) is greater than the value of Chi Square with p degrees of freedom. The rejection of the null hypothesis indicates the importance of the ARCH process.

Table 4 contains outputs of the ARCH test in our sample. Here we use the p^{th} order linear declining weight process and let the conditional variance as follows

$$V_t = \alpha_0 + \alpha_1 \sum_{i=1}^p W_i \epsilon_{t-i}^2 \quad (8)$$

where w_i denote the linearly declining weights. In this analysis, $p = 1, 4$, and 8 . It is found that almost all tests are significant at the 5% level. Thus the more efficient estimates of the hedge ratios are obtained using the maxi-

Table 4
ARCH Process Test

		α_0	α_1	X^2
SP1W	ARCH p = 1	0.000021 (5.6460)	0.131267 (1.8203)	3.29**
	ARCH p = 4	0.000016 (3.6777)	0.345909 (3.0118)	8.74**
	ARCH p = 8	0.000012 (2.5789)	0.498208 (3.5881)	12.16**
SP2W	ARCH p = 1	0.000030 (3.1279)	0.214570 (2.1556)	4.52**
	ARCH p = 4	0.000018 (1.8025)	0.537327 (3.8137)	12.80**
	ARCH p = 8	0.000005 (0.7226)	0.702690 (6.6134)	29.67**
SP4W	ARCH p = 1	0.000066 (2.2314)	0.23489 (0.1575)	0.03
	ARCH p = 4	0.000030 (3.1159)	0.171388 (1.8472)	3.31**
	ARCH p = 8	0.000017 (1.2765)	0.463416 (2.0518)	3.99**

** reject OLS with one tailed 5% test
(numbers in parentheses are t-statistics)

maximum likelihood method by taking into account the variance which follow the ARCH process. Table 5 lists the results generated. From the results one can observe that the estimated hedge ratios are generally greater than the OLS estimates. In addition, the t-values for all the hedge ratios estimated from the ARCH model are higher than those from the OLS. The higher t-value indicates that the efficiency of the hedge ratio estimation has been improved due to lower standard deviation of the coefficient.

[IV]. Application of The Bivariate ARCH Model

The bivariate ARCH procedure developed in Kraft and Engle (1982) can be applied to estimate the optimal hedge ratio when the joint distribution of returns to the cash position and futures position are unstable over time. Before we illustrate the application of this technique, let's briefly review the concept of the anticipatory hedge. Peck (1975) and Rolfo (1980) indicated

Table 5
Hedge Ratio Estimation from Univariate ARCH Model

	<u>Initial OLS</u>	<u>ARCH p=1</u>	<u>ARCH p=4</u>	<u>ARCH p=8</u>
SP1W	0.9026 (55.9572)	0.9248 (64.5079)	0.9169 (58.6565)	0.9225 (56.4948)
SP2W	0.9300 (45.4089)	0.9648 (55.1147)	0.9552 (52.3374)	0.9617 (58.9978)
SP4W	0.9039 (35.3420)	0.9039 (36.3044)	0.9355 (41.9789)	0.9422 (42.1185)

(numbers in parentheses are t-statistics)

that with an anticipatory hedge one is not hedging against price movement per se, but against movements in unanticipated price changes. Since the price expectations have been incorporated in making the cash market decision, the objective of the hedger is to avoid surprises or unanticipated price changes. In terms of returns, the unanticipated returns to the cash position and the futures position can be defined as

$$\epsilon_{s,t} = R_{s,t} - E(R_{s,t} | \Phi_{t-1}), \text{ and}$$

$$\epsilon_{f,t} = R_{f,t} - E(R_{f,t} | \Phi_{t-1})$$

respectively,

The anticipated surprises are assumed to be zero. The conditional covariance matrix to be used in the hedge ratio estimation can then be obtained from the bivariate ARCH procedure. Assuming that conditional variances of returns to the cash position and futures position follow a p th order linearly declining weight ARCH process, the three-equation model to be estimated is then expressed as

$$E(\epsilon_{s,t} | \Phi_{t-1})^2 = \alpha_0 + \alpha_1 \sum_{i=1}^p W_i \epsilon_{s,t-i}^2 \quad (9)$$

$$E(\epsilon_{f,t} | \Phi_{t-1})^2 = \beta_0 + \beta_1 \sum_{i=1}^p W_i \epsilon_{f,t-i}^2 \quad (10)$$

and

$$E(\epsilon_{s,t} \epsilon_{f,t} | \Phi_{t-1}) = \rho [E(\epsilon_{s,t} | \Phi_{t-1})^2 E(\epsilon_{f,t} | \Phi_{t-1})^2]^{1/2} \quad (11)$$

where w_i denote the linearly declining weights and ρ denotes the correlation between expected returns to cash and futures positions (the correlation is assumed to be constant over time). As long as the ρ is less than one in absolute

value, the covariance matrix of returns will be positive definite. Moreover, another assumption is employed in the empirical investigation. That is, we assume that returns to the cash position and the futures position follows a random walk process. Hence the surprises in returns become:

$$\epsilon_{s,t} = R_{s,t} - R_{s,t-1}, \text{ and}$$

$$\epsilon_{f,t} = R_{f,t} - R_{f,t-1}$$

where $R_{s,t-1}$ and $R_{f,t-1}$ denote the returns to the cash position and the futures position at $t-1$, respectively.

Table 6 contains the estimated results from Eq. (9) through Eq. (11). For the one week and two-week hedges the ARCH test is significant with $p = 8$. For the four-week hedge the test is significant with $p = 4$.

After we have constructed the conditional covariance matrix, the time-varying hedge ratio can be obtained from:

$$h_t = \frac{E(\epsilon_{s,t}\epsilon_{f,t} | \Phi_{t-1})}{E(\epsilon_{f,t} | \Phi_{t-1})^2} \quad (12)$$

The effectiveness of this time-varying adjustment strategy is examined in section III.

[V]. *Application of The Generalized Changing Parameter Model*

In addition to the possibility of random movement, changes in the hedge ratio may be determined by variables not included in the traditional hedge model. Specifically, if the nonstationarity of the hedge ratio is determined by

$$h = h_0 + \sum_{i=1}^k \alpha_i X_i \quad (13)$$

Table 6
Results for Bivariate ARCH Model

SP1W ARCH p=8

$$V_t(s) = 0.00029 + 0.63288 \sum_{i=1}^8 \left(\frac{9-i}{36} \right) V_{t-i}(s)$$

(2.2822) (5.3965)

$$V_t(f) = 0.00029 + 0.68565 \sum_{i=1}^8 \left(\frac{9-i}{36} \right) V_{t-i}(f)$$

(2.0155) (6.4662)

$$V_t(s,f) = 0.98195 [V_t(s)V_t(f)]^{1/2}$$

(11.1447)

SP2W ARCH p=8

$$V_t(s) = 0.00063 + 0.42800 \sum_{i=1}^8 \left(\frac{9-i}{36} \right) V_{t-i}(s)$$

(2.1921) (4.1597)

$$V_t(f) = 0.00053 + 0.49535 \sum_{i=1}^8 \left(\frac{9-i}{36} \right) V_{t-i}(f)$$

(1.8254) (5.1485)

$$V_t(s,f) = 0.96820 [V_t(s)V_t(f)]^{1/2}$$

(8.0646)

SP4W ARHC p=4

$$V_t(s) = 0.00057 + 0.4998 \sum_{i=1}^4 \left(\frac{5-i}{10} \right) V_{t-i}(s)$$

(0.8384) (6.8240)

$$V_t(f) = 0.00051 + 0.5099 \sum_{i=1}^4 \left(\frac{5-i}{10} \right) V_{t-i}(f)$$

(0.6720) (8.3281)

$$V_t(s,f) = 0.9973 [V_t(s)V_t(f)]^{1/2}$$

(9.0163)

(numbers in parentheses are t-statistics)

then h can be regarded as a decision variable instead of a fixed coefficient. By substituting this relationship directly into the original estimation, Eq. (1), we get

$$R_{st} = a_0 + h_0 R_{ft} + \sum_{i=1}^k \alpha_i X_i R_{ft} + \epsilon_t \quad (14)$$

Moreover, if the relationship for h includes a random factor, u_t , and h_t is determined by

$$h_t = h_0 + \sum_{i=1}^k \alpha_i X_i + u_t \quad (15)$$

then the final estimating equation becomes

$$R_{st} = a_0 + h_0 R_{ft} + \sum_{i=1}^k \alpha_i X_i R_{ft} + w_t \quad (16)$$

where

$$w_t = u_t R_{ft} + \epsilon_t$$

In this case changes in the hedge ratio can be attributed to the systematic component and/or a random shock. If the variance of u_t is statistically significant, then the more complicated nature of the error term results in inefficient OLS estimates. A more sophisticated procedure, such as the generalize least squares technique as illustrated in Part [I], could be performed to improve the efficiency. On the other hand, if all of the α parameters are all insignificant, Eq. (16) can be reduced to the pure random coefficient model. Finally, if both the alphas and the variance of u_t are all insignificant, then model Eq. (16) is nothing but the fixed coefficient model Eq. (1).

To test this generalized changing parameter model we implement the portfolio hedging strategies using options on individual stocks, options on the S&P 100 index, and options on the S&P 500 index futures.

It is well known that in addition to the underlying security price, several other variables also have an important effect on option prices. These variables include the exercise price, time to expiration of the option, the risk-free interest rate, and the volatility of the underlying security. Accordingly, in performing a pooled time-series and cross-section regression to estimate the options hedge ratio, we specify changes in the hedge ratio to be determined by

$$h_t = h_0 + \alpha_1 (E/S) + \alpha_2 (T) + \alpha_3 (\Delta C) + \alpha_4 (R) + \alpha_5 (\sigma) + u_t \quad (17)$$

where E/S = ratio of the exercise price to the underlying security price,

T = option's time to expiration in annual basis,

ΔC = change in call option prices,

R = T-Bill rate annualized, and

σ = annual volatility of the underlying security.²

Since the relationship between the option price and its underlying security price is nonlinearly related, we also include a ΔC in Eq. (17). Thus the estimating equation is augmented as

$$\begin{aligned} \Delta S_t = & \alpha_0 + h_0 \Delta C + \alpha_1 (E/S) \Delta C + \alpha_2 (T) \Delta C + \alpha_3 \Delta C^2 + \alpha_4 (R) \Delta C \\ & + \alpha_5 (\sigma) \Delta C + w_t \end{aligned} \quad (18)$$

where

$$w_t = u_t \Delta C_t + \epsilon_t, \text{ and}$$

$$\Delta S = \text{changes in the underlying security price.}$$

² The weighted implied standard deviation (ISD) technique suggested in Whaley (1982) is employed to estimate the volatility of the security.

The randomness of h_t is examined by performing the following regression

$$w_t^2 = b_0 + b_1 c_t^2 \quad (19)$$

where

$$c_t = \Delta C_t - \Delta C$$

The procedure is similar to the one we explained in Part [I] of this section.

The results of the stability test in the last row of Table 7A reveal that none of the hedge ratios for S&P 100 options, S&P 500 futures options, and AT&T options significantly fluctuates with random shock. Hence Eq. (14) is an appropriate specification for the option hedge ratio in those samples. However, the same results do not hold in Table 7B for IBM options, Eastman Kodak options, and American Express options. Thus the GLS technique is performed to estimate Eq. (18) for these three samples. Almost all of the t-values for coefficient estimates in Eq. (17) presented in Tables 7A and 7B are significant at 5% level. This finding suggests that the application of the regression method to estimate the optimal hedge ratio for the option hedge should incorporate more factors than should the futures hedge.

III. EFFECTIVENESS OF HEDGING

The effectiveness of futures and options as hedging instruments depend upon the degree to which the price behavior of the hedging instruments mimics that of the underlying cash position. Currently, the most commonly used measure of hedging effectiveness is the percentage reduction in variance of return from the unhedged position. It can be measured from the square of the correlation coefficient between cash price changes and futures price changes, R^2 . This measure was first derived in Johnson (1960) and is applied

Table 7A
Results for Generalized Changing Parameter Model
[Equation (18) and Equation (19)]

	AT&T	Option on S&P 100	Option on S&P 500 Futures
a_0	0.0872 (4.5985)	0.6515 (11.2440)	0.4084 (9.3309)
α_1	-1.4644 (-1.6398)	-7.8742 (-10.9716)	-16.5810 (-16.8247)
α_2	4.7503 (9.6259)	8.5331 (11.4083)	17.1990 (19.5381)
α_3	0.4934 (1.6950)	-1.1570 (-2.5342)	0.2088 (1.2782)
α_4	-0.1231 (-1.7492)	-3.0923 (-2.8230)	0.0272 (2.3169)
α_5	-0.1950 (-2.1839)	0.1615 (4.4575)	0.1731 (3.5250)
R^2	0.6824	0.6929	0.8700
b_0	0.1168 (10.2284)	2.2746 (13.2671)	0.8649 (7.5492)
b_1	0.0268 (0.5985)	-0.0233 (-0.8721)	0.0030 (0.1039)

(numbers in parentheses are t-statistics)

Table 7B

Results for Generalized Changing Parameter Model
[Equation (18) and Equation (19)]

	IBM	Eastman Kodak	American Express
a_0	0.3711 (6.7073)	0.0203 (0.4260)	0.2032 (3.5434)
α_1	-4.8702 (-10.8560)	-4.3235 (-3.7510)	-3.3689 (-3.7997)
α_2	6.6670 (13.7831)	5.4369 (4.4850)	2.4876 (3.8709)
α_3	-0.1396 (-0.7581)	0.7126 (1.8251)	0.2256 (0.52782)
α_4	-0.0271 (-2.5922)	-0.0123 (0.2996)	-0.0149 (-0.8794)
α_5	—	—	0.2367 (2.5983)
R^2	0.8616	0.5964	0.5628
b_0	1.4819 (5.4947)	0.7056 (5.2279)	1.2302 (8.3779)
b_1	0.1496 (5.9038)	0.2922 (6.7841)	0.0641 (3.2848)

(numbers in parentheses are t-statistics)

by Ederington (1979) in examining the portfolio performance in financial futures markets. It is shown that the higher the correlation between cash and futures price changes, the more effective the futures hedge is. By performing a simple regression, the coefficient of determination from regressing R_s on R_f represents the hedging effectiveness. However, in most of the hedging practices, the error terms were often serially correlated. One problem with the presence of autocorrelation is that the R^2 statistic will be biased upward, and the bias will exaggerate the merits of hedging. Under this circumstance, the GLS technique is generally employed to deal with this problem. Note that the R^2 statistic from the GLS no longer represents the percentage reduction in the variance of the original dependent variable. Therefore, the measure of hedging effectiveness should be revised as:

$$HE' = 1 - \frac{\sum_{i=1}^T (R_{st} - \hat{R}_{st})^2}{\sum_{i=1}^T (R_{st} - \bar{R}_s)^2} \quad (20)$$

where \hat{R}_{st} denotes the predicted value of the dependent variable using GLS estimates as the model's parameters, i.e.,

$$\hat{R}_{st} = \hat{\alpha}_{GLS} + \hat{h}_{GLS} R_{ft}$$

By substituting the GLS estimates into the original regression equation the revised measure can appropriately gauge the percentage reduction in the variance that is achieved by undertaking a hedging strategy.

Furthermore, while hedging reduces the risk of the cash position, it may also affect the expected return of the cash position. The variance comparison focuses on only the aspect of risk reduction but ignores the issue of expected return. For this reason, it is inconsistent with the rationale behind

the portfolio selection approach. Consequently, Nelson and Collins (1985) suggested the application of the Sharpe performance measure to evaluate the hedging performance. The effectiveness of a hedging strategy is represented by a single index:

$$\theta = \frac{R_h}{\sigma_h}$$

where R_h and σ_h denote the expected return and the standard deviation of the hedged portfolio, respectively. Howard and D'Antonio (1984) applied the same concept and redefined the hedging effectiveness measure as the ratio of the return-risk relative of the hedged portfolio to the return-risk relative of the unhedged portfolio, i. e.,

$$\Omega = \frac{\theta}{(R_s - r)/\sigma_s}$$

Since Ω employs the unhedged position as a benchmark, it is particularly applicable in the comparison of the hedging performance among different cash portfolios.

It is important to point out that only quadratic utility functions or normal return distributions are appropriately described by the means and variances. Hence, the use of the first two moments is quite restrictive. If probability distributions of returns are asymmetric, then the third moment should be considered in the analysis. In particular, the holding period return distribution of the options hedge is known to be skewed. Therefore, when a portfolio is augmented by an option hedging strategy the classic mean-variance criterion will not suffice. Below, we will examine the hedging effectiveness of static as well as dynamic hedging strategies in terms of different performance measures.

Table 8A presents the return characteristics of unhedged positions, the conventional naive hedged position, and the minimum-variance hedged positions. While both the naive hedge and minimum-variance hedge can eliminate a significant proportion of the variability of unhedged cash positions, they can also reduce the average returns to the hedged positions. This result by no means conflicts with the hedging design. Table 8B presents the summary indexes of hedging effectiveness; they include the conventional measure R^2

Table 8A
Hedging Effectiveness of Conventional Hedging Strategies

		<u>R</u>	<u>σ</u>	<u>SK</u>	<u>β</u>
SP2W	Unhedged	0.1693	0.1472	0.6825	1.0000
	N-Hedge	0.0452	0.0384	-0.5797	-0.0445
	H*-Hedge	0.0573	0.0352	-0.2641	0.0545
SP2W	Unhedged	0.1702	0.1532	1.4718	1.0000
	N-Hedge	0.0475	0.0342	-0.0471	-0.0277
	h*-Hedge	0.0561	0.0322	0.0078	0.0442
SP4W	Unhedged	0.1711	0.1575	1.5616	1.0000
	N-Hedge	0.0456	0.0339	-1.3626	-0.0670
	h*-Hedge	0.0577	0.0297	-1.4157	0.0355

Unhedged = unhedged position,

N-hedge = naive one-to-one hedge,

h*-hedge = minimum-variance hedge,

R = the annualized average return,

σ = the annualized standard deviation of return,

SK = the skewness of the return distribution.

Table 8B
Hedging Effectiveness of Conventional Hedging Strategies

		R^2	θ	Ω
SP1W	Unhedged	0.0000	1.1501	1.0000
	N-Hedge	0.9319	1.1777	1.0240
	h*-Hedge	0.9028	1.6278	1.4154
SP2W	Unhedged	0.0000	1.1110	1.0000
	N-Hedge	0.9502	1.3889	1.2501
	h*-Hedge	0.9538	1.7422	1.5680
SP4W	Unhedge	0.0000	1.0863	1.0000
	N-Hedge	0.9537	1.3451	1.2382
	h*-Hedge	0.9644	1.9428	1.7885

R^2 = the percentage reduction in cash portfolio variance,

θ = return-risk relative = R/σ ,

Ω = Howard-D'Antonio hedging effectiveness measure.

as well as θ and Ω . A close examination of Table 8B reveals that the minimum-variance hedging strategy is superior to the naive hedging strategy in terms of return, risk, skewness, or return-risk relative. In addition, the long hedging horizon seems to outperform the short hedging horizon. Also, the longer the hedging horizon the better the minimum-variance hedge.

Tables 9A and 9B present the return characteristics of dynamic hedging strategies. Comparing the results in Table 9B to those in Table 8B, we observe that the values of R^2 , θ , and Ω in Table 9B are all higher than those in Table 8B. This finding strongly supports the superiority of more sophisticated dy-

Table 9A
Hedging Effectiveness of Dynamic Hedging Strategies

		<u>R</u>	<u>σ</u>	<u>SK</u>	<u>β</u>
SP1W	B-Hedge	0.0617	0.0250	0.3368	0.0387
	E-Hedge	0.0574	0.0362	-0.2369	0.0489
SP2W	B-Hedge	0.0583	0.0253	0.3185	0.0329
	E-Hedge	0.0556	0.0262	0.6973	0.0000
SP4W	B-Hedge	0.0572	0.0361	-1.3687	0.0000
	E-Hedge	0.0612	0.0244	-0.0608	0.0000

B-Hedge = the dynamic hedge with hedge ratio estimated from the Bayesian model,

E-Hedge = the dynamic hedge with hedge ratio estimated from the bivariate ARCH model,

R = the annualized average return,

σ = the annualized standard deviation of return,

SK = the skewness of the return distribution.

Table 9B
Hedging Effectiveness of Dynamic Hedging Strategies

		<u>R²</u>	<u>θ</u>	<u>Ω</u>
SP1W	B-Hedge	0.9712	2.4680	2.1459
	E-Hedge	0.9395	1.5856	1.3787
SP2W	B-Hedge	0.9727	2.3043	2.0741
	E-Hedge	0.9708	2.1221	1.9101
SP4W	B-Hedge	0.9475	1.5845	1.4586
	E-Hedge	0.9760	2.5082	2.3089

B-Hedge = the dynamic hedge with hedge ratio estimated from the Bayesian model,

E-Hedge = the dynamic hedge with hedge ratio estimated from the bivariate ARCH model.

dynamic hedging strategies over conventional approaches. Moreover, by comparing the Bayesian adjustment strategy and the bivariate ARCH adjustment strategy, we can see that the former has higher average returns for shorter hedging periods while the latter behaves with a lower standard deviation in the longer term of the hedging horizon. The comparison implies that the Bayesian adjustment strategy seems to outperform the bivariate ARCH adjustment strategy for the one-week and two-week hedges (but not for the longer term hedge). Finally, in all cases the values of skewness in Table 9A are higher than those in Table 8A. This indicates an additional relative advantage provided by dynamic hedging strategies.

IV. SUMMARY AND CONCLUSIONS

With the changing nature of the real world it is unrealistic to assume that the hedge ratio estimated from historical data over a long period is constant. The dynamic nature of the hedge ratio presents a practical problem in performing OLS to compute the optimal hedge ratio. Five econometric methods are employed to deal with the problem of dynamic adjustment. In addition, these methods are applied to examine the dynamic hedging potentials for stock portfolios. In particular, while previous studies centered on only evaluates hedging performance based on the return-risk relative as well as the skewness of the hedged return. The results show that the average returns and the skewness coefficients of the hedged returns from dynamic hedging strategies are higher than the naive hedge and the minimum-variance hedge. The risk reduction potential of the dynamic hedging strategies is also higher than the other two approaches. Accordingly, the return-risk relatives from the dynamic strategies are substantially higher than the static hedging strategies. These findings give strong support to the superiority of the more sophisticated dynamic hedging strategies.

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最適避險比率的動態調整

林 筠

摘 要

「避險」乃是期貨市場及選擇權市場最主要的功能之一，避險者只要持有適當的期貨合約或選擇權合約，便能減少其現貨部位所面臨的價格波動風險；至於避險成效的大小，則取決於避險比率是否適當而定。因此如何決定最適避險比率便成為學術界及實務界一個重要的課題。在過去避險的實証研究中，大多只著重於如何經由歷史資料，觀察期貨價格與現貨價格變動的相關性，並據以得出一個靜態的最適避險比率。事實上在一個充滿變數的經濟社會中，此一靜態避險比率之穩定性，一直受到相當大質疑。因此本研究乃應用計量分析方法，提出五種最適避險比率之動態調整策略，並以 S&P500 股價指數期貨及選擇權作為避險工具，說明如何應用本文所建議之動態調整策略，以增進股票投資組合之避險成效。尤其要強調的是本文在評估各種避險策略之成效時，除了比較一般文獻上新採用的報酬率變異數減少百分比(R^2)外，並比較各種避險策略在平均報酬率以及報酬率機率分配在偏度上的差異，以期提供更廣泛而公平的評估基礎。