

LIQUIDITY FOR MARKING-TO-MARKET¹

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ABSTRACT

Futures trading is characterized by marking-to-market. Each trading day the losing party, be it the buyer or seller of the contract, must pay the full amount of the futures price change occurring that day. When prices move consistently against a trader's positions, marking-to-market can cause liquidity problem. Such a trader may then be forced out of the desired contracts. This paper applies a principle of cash management to solve the liquidity problem. An optimal initial working capital is derived using the theory of stochastic processes. Whenever exhausted, it would be replenished from a better-earning source to the initial optimal level. This approach differs from a current study, as it allows traders to trade indefinitely.

¹ The main idea of this paper was previously presented at the 1988 annual conference of the Financial Management Association in New Orleans, under the title of "Qualifying Futures Traders — A Scientific Approach." I would like to thank the participants of that seminar.

INTRODUCTION

Futures trading is characterized by marking-to-market.² Each trading day the losing party, be it the buyer or seller of the futures contract, must pay the full amount of the futures price change occurring that day.³ The requirement of marking-to-market is designed to protect the contract from default. After cash settlement of each day, the contract is essentially rewritten. However, it may not be the case that the trader on the losing side is ready to liquidate the contract. When prices move consistently against a trader's position, marking-to-market can cause liquidity problem.⁴ Such a trader may then be forced out of the desired contracts although he or she is not prepared to do so.

The liquidity problem is aggravated by the inherent high leverage in futures trading. The initial investment for a futures trader, in the form of initial margin, is a small fraction of the value of the underlying asset.⁵ Therefore, a single day's change in the price of the underlying asset may represent a 50% change or more, on the initial investment. Due to the significant risk, brokerage firms tend to evaluate a customer's financial strength before the customer can qualify as a trader. Usually, a customer's total net worth and annual income should exceed a minimum amount set by the brokerage firms.

In order to be well-prepared for trading futures, Kolb, Gay and Hunter

2 Marking-to-market is also required in equity short sales and options writing. Although this paper uses futures to illustrate the liquidity problem associated with the requirement, the proposal can certainly be modified to apply to those markets.

3 Brokers have to pay to the clearinghouse to cover losses sustained by their customers. The customers, in turn, have to deposit additional funds in order to maintain the value of their accounts with the brokerage firms at the specified initial margins.

4 The problem associated with marking-to-market has been said to be one of the main reasons that encourage commercial banks to offer now successful custom-made futures contracts, see Brown and Smith (1988).

5 Usually, the initial margin is less than 10% and may be as low as 5% of the futures price for stock index futures.

(1985) applied the well-known gambler's ruin problem to derive a "liquidity pool" to meet margin calls. However, their model has no concern about the sustenance of futures holding. When the pool is drained,⁶ no further suggestion is made. In this paper, a new proposal is made that will ensure the sufficiency of the pool. The paper is organized as follows: in Section I, the concept initiated by Frenkel and Jovanovic (1980) is borrowed to derive an optimal initial working capital that would be replenished from a better-earning source, whenever the working capital drops below a specified lower bound.⁷ To determine how many times the replenishment may occur, Section II brings in the expected time by which the initial working capital would reach the lower bound, from the theory of stochastic processes. All results are then combined with the planned trading horizon which a customer has in mind to determine the minimum required net worth for the customer to sustain desired futures contracts. Section III concludes the paper by suggesting other applications of the model. In particular, the minimum required net worth may also serve as a basis to compute service charge for innovative financial institutions to provide interest rate swap.

I. THE OPTIMAL WORKING CAPITAL

Assume the dynamics for the futures price follow:

$$dF = \alpha(F)dt + \sigma(F)dZ, \quad (1)$$

where

F is the futures price,

$\alpha(F)$ is the drift term of the futures price dynamics,

$\sigma(F)$ is the instantaneous standard deviation of the dynamics,

⁶ The probability for this to happen is positive, see Feller (1968).

⁷ Cox and Kuo (1987) outlines a framework for two-bound solution without deriving the specific closed-form as done in this paper. Market observations suggest that one bound solution seems to be a more common practice.

dZ is the increment of a standard Wiener Process.

Whenever an investor enters into a futures contract for hedging or speculation purpose, the limitation of the investor's net worth is vulnerable to the randomness of futures price fluctuations. Our task is to develop a solution to the liquidity problem. Since we are concerned about the trader's risk exposure in the future, we should take into account the dynamics for his or her net worth. Suppose $F(0;d)$ is the investor's contracted futures price at time zero with delivery date d . And $F(1;d)$, $F(2;d)$, . . . , are the subsequent market futures prices for the same maturity. Then, the contribution of variation margins to the investor's net worth for one single trading day is:

$$X_t = F(t;d) - F(t-1;d),$$

starting from $t = 1$, where

X_t = the variation margins that may trigger margin call.

Consequently, the contribution after t trading days is the accumulation:

$$\sum_{\gamma=1}^t X_{\gamma} = F(t;d) - F(0;d).$$

Now, assume that the investor sets up an initial working capital, $W(t=0)$, to take care of marking-to-market. After t periods, the level of working capital will be:

$$W(t) = W(0) + \sum_{\gamma=1}^t X_{\gamma} = W(0) - F(0;d) + F(T;d). \quad (2)$$

where

$W(t)$ is the level of the working capital at time t .

Also assume that marking-to-market occurs continuously through time so that the net value of the futures contract always remains zero. Since the distribution of futures prices of equation (1) can be characterized by:

$$F(t;d) = F(0;d) + \alpha(F)t + \sigma(F)Z(t), \quad (3)$$

substituting equation (3) for $F(t;d)$ in equation (2), the distribution of working capital can be characterized by:

$$\begin{aligned} W(t) &= W(0) + \alpha(F)t + \sigma(F)Z(t), \text{ or} \\ dW &= \alpha(F)dt + \sigma(F)dZ. \end{aligned} \quad (4)$$

Thus, equation (4) shows that changes in the working capital account follow the dynamics for the futures price. Note that the dynamics ignore the maximum daily price change imposed on most commodities and financial futures. It is contended that the imposition of daily price limits is neither a necessary nor a sufficient condition to eliminating an investor's risk of liquidity exhaustion. The market would move eventually toward equilibrium although the exchange's policy might delay the process. An ill-prepared investor would be "ruined" sooner or later if the market moves against his or her position.⁸

The working capital can be deposited in a checking account.⁹ A large amounts of cash deposited in the account apparently minimize liquidity risk but also imply opportunity lost. Part of the capital may be put to better use. Therefore, this is recognized as a cash management problem and it is

⁸ In the history of the Chicago Board of Trade, no trader has suffered from default since the clearinghouse of the exchange guarantees the contract performance.

⁹ When a trader receives a margin call, he or she writes a check to settle the losses.

possible to derive an optimal initial working capital by minimizing the cost of financial management. In the following, the concept initiated by Frenkel and Jovanovic (1980) is borrowed for the purpose. In their seminal paper, Frenkel and Jovanovic proposed to consider two types of cost: the first being foregone better earning opportunities for maintaining a working capital account; and the second being the cost of adjustment that had to be made to restore the account back to its optimal level, whenever the account dropped below an undesirable lower bound which is assumed to be zero without loss of generality.

The working capital earns a constant checking account rate, r . The opportunity cost for setting up the account can be measured by the differences between returns on better-earning sources, and r . For simplicity, the investor is assumed to have chosen one single better-earning source. A candidate for this probably better rate of return is $\mu(d)$, the yield-to-maturity on a riskless Treasury Issue maturing at the same time, d , as the futures contract. $\mu(d) = [\log K - \log P(0;d)]/d$, assuming continuous compounding, where K is the face value, and $P(0;d)$ is the current price of the Treasury Issue.¹⁰

Therefore, the present value of the instantaneous foregone better earnings for time t can be expressed as:

$$[\mu(d) - r] W(t) e^{-rt} dt.$$

Denote by $h(W,t)$ the probability that the working capital $W(t)$ will not reach zero prior to the period t . That is, $h(W,t)$ is the conditional probability density of $W(t)$, given $W(s) > 0$, $0 < s < t$. The present value of the expected foregone earnings up to the first adjustment is:

$$J_1[W(0)] = \int_0^d [\mu(d) - r] e^{-rt} \left\{ \int_{W=0}^{\infty} W h(W,t) dW \right\} dt. \quad (5)$$

¹⁰ We may view this portion of better earning net worth as in the "semi-liquid" form, as opposed to the working capital which is in the liquid form.

In addition to the opportunity cost specified by equation (5), the working capital management also involves cost of adjustment. The adjustment of the working capital account is necessary because there is a positive probability that the account would reach the lower bound, zero, due to the stochastic nature of futures price dynamics.

Assume for each adjustment, the trader arranges a refinancing so as to restore the working capital to the optimal level $W(0)$. By so doing, a fixed cost, c , per adjustment is incurred (such as a broker's fee). According to Frenkel and Jovanovic, the cost of adjustment also depends upon the frequency of adjustment and thus, could be expressed by:

$$J_2[W(0)] = \int_0^d e^{-rt} \{c + G[W(0),t]\} f[W(0),t] dt, \quad (6)$$

where

$G[W(0),t]$ is the present value of total expected cost,

$f[W(0),t]$ is the probability that $W(t)$ reaches zero at t .

Note that by definition, the total expected cost, $G[W(0),t]$, is the combination of equations (5) and (6):

$$G[W(0),t] = J_1[W(0)] + J_2[W(0)] .$$

The equation can be solved for $G[W(0),t]$ by numerical methods. However, a closed-form solution has the handy, easy-to-use advantage. This advantage is important as there are numerous contracts available for a trader's hedging or speculation purposes. A complex numerical procedure may not be practical for each contract in the trader's portfolio.

For the purpose of obtaining a closed-form solution, the horizon in equations (5) and (6) is extended to infinity, i. e., $0 \leq t < \infty$.¹¹ In addition,

if the dynamics for the futures price, as of equation (1), are further assumed to follow the Brownian motion process, that is, the two parameters $\alpha(F)$ and $\sigma(F)$ are fixed constants,¹² then the total expected cost can be rewritten as (See Appendix I):

$$G[W(0)] = \left[\frac{\mu(d)}{r} - 1 \right] \left[\frac{W(0)}{(1-\Gamma)} - \frac{\alpha(F)}{r} \right] + \frac{\Gamma c}{(1-\Gamma)}, \quad (7)$$

where Γ is the Laplace transform of $f[W(0),t]$ such that

$$\Gamma = \int_0^{\infty} e^{-\tau t} f[W(0),t] dt.$$

Minimizing $G[W(0)]$ with respect to the initial working capital $W(0)$ yields the necessary condition for optimality:

$$\begin{aligned} \frac{\delta G}{\delta W(0)} = & \left[\frac{\mu(d)}{r} - 1 \right] \left[\frac{1}{(1-\Gamma)} + \frac{W(0)}{(1-\Gamma)^2} \frac{\delta \Gamma}{\delta W(0)} \right] \\ & + \left[\frac{c}{(1-\Gamma)} \frac{\delta \Gamma}{\delta W(0)} + \frac{\Gamma c}{(1-\Gamma)^2} \frac{\delta \Gamma}{\delta W(0)} \right] = 0. \end{aligned}$$

Therefore,

$$\left[\frac{\mu(d)}{r} - 1 \right] (1-\Gamma) + \left\{ \left[\frac{\mu(d)}{r} - 1 \right] W(0) + c \right\} \frac{\delta \Gamma}{\delta W(0)} = 0. \quad (8)$$

Equation (8) can then be solved for the optimal $W(0)$ (See Appendix II):

11 With the brought-in "inflated" cost which apparently is positive, the resulting optimal working capital may be considered conservative.

12 This is equivalent to saying that, whenever there are changes in the two parameters, the optimal working capital should be re-computed.

$$W(0) = \frac{2rc}{R[\mu(d) - r]}, \quad (9)$$

where

$$R = \frac{1}{\alpha(F)^2} \{ [\alpha(F)^2 + 2r\sigma(F)^2]^{1/2} - \alpha(F) \}.$$

Equation (9) is the optimal working capital to be set up when a futures contract is initiated. Due to the random nature of futures price movements, there is a positive probability that the working capital account will be exhausted. When this occurs, a transaction in the financial market is assumed to be undertaken to replenish the account back to $W(0)$. As can be seen, it requires the estimation of the two parameters in the dynamics for the price only. Therefore, it is a handy tool to compute the liquidity needs.

II. THE MINIMUM REQUIRED NET WORTH

For the purpose of determining the minimum net worth a trader should prepare for desired futures holding, it is essential to know how many times the trader may have to replenish the working capital account. The problem can be solved if the expected time by which the account would be exhausted is known.

When the dynamics for the working capital, as of equation (4), follow the Brownian motion process, the expected time to the exhaustion of an initial amount $W(0)$ as defined by:

$$v[W(0)] = E[T|W(0)],$$

where $W(0) > 0$, and

T is the first time $W(t)$ is zero,

satisfies the differential equation:

$$-1 = \alpha(F) \frac{\delta v}{\delta W(0)} + \frac{1}{2} \sigma(F)^2 \frac{\delta^2 v}{\delta W(0)^2},$$

subject to the initial condition: $v(0) = 0$.

The solution to this differential equation is (see Karlin and Taylor, 1981, pp. 197):

$$\begin{aligned} v[W(0)] = & 2 \left\{ u[W(0)] \int_{W(0)}^{\infty} [S(\infty) - S(\Phi)] m(\Phi) d\Phi \right. \\ & \left. + [1 - u[W(0)]] \int_{\Phi=0}^{W(0)} S(\Phi) m(\Phi) d\Phi \right\}, \end{aligned} \quad (10)$$

where

$$S(y) = \int_0^y s(\Omega) d\Omega,$$

$$s(\Omega) = \exp \left\{ - \int_0^{\Omega} \frac{2\alpha(z)}{\sigma(z)^2} dz \right\},$$

$$m(z) = \frac{1}{s(z)\sigma(z)^2},$$

$$u[W(0)] = S(W(0))/S(\infty).$$

This expected time to capital exhaustion, if combined with the optimal initial working capital from the previous section, provides tools for the futures trader qualifying purpose. Take an example of a hedger with planned hedging horizon d ,¹³ $d/E(T)$ represents the approximate number of times the hedger has to refinance the optimal working capital account. Then, the hedger would

need to prepare a minimum net worth no less than W^* in liquid or semi-liquid form:

$$W^* \geq X(F) [I + W(0) + [d/E/T] (W(0) + c)], \quad (11)$$

where

I is the initial margin,

$X(F)$ is the number of futures contracts.

From expression (11), it can be seen that the minimum net worth would cover the initial margin and the margin calls that may occur during his or her desired futures holding periods. If such a minimum amount is prepared, hedgers and speculators will be able to concentrate on their spot and/or futures portfolio management without being distracted from the management of margin accounts. Note that (11) can be easily expanded to incorporate multiple contracts if written in matrix form. Of course, $E(T)$ and $W(0)$ must be individually computed for each contract.

III. CONCLUSION

This paper proposes a new method to deal with the liquidity problem associated with the requirement of marking-to-market. The method remedies the deficiency of Kolb, Gay and Hunter (1985) proposal that can not ensure the sustenance of contract holdings. By making some conservative assumptions, a closed-form solution of an optimal working capital for trading a futures

¹³ Hedgers usually have a well-defined hedging period. On the other hand, some speculators may have in mind a particular direction and magnitude of price movements. Before that happen, they may not like to be forced out of speculating futures positions.

contract is derived that requires estimation of two parameters only. Combined with information such as the expected time by which the initial working capital would be exhausted, and a trader's planned horizon, the trader can determine the minimum net worth he or she should prepare for those tradings that require marking-to-market. Especially for hedgers in futures markets, being able to stay in the planned futures positions is the key to effective hedging. Financial institutions, such as insurance companies and pension funds, are not permitted to trade futures as much as desired. The concerns of regulators are leverage and risks involved in futures trading that may be detrimental to the institutions' daily cash flows. By applying the approach to determine the relevant liquidity risk, the issue may become less controversial.

When futures traders prefer to concentrate on their trading activities to being distracted from the liquidity concerns of marking-to-market, it is desirable for innovative financial institutions to provide certain interest rate swap services in which the financial institutions take care of marking-to-market and charge the traders a fixed rate on possible financing involved.¹⁴ The minimum required net worth as derived in the previous section would serve as a basis for computing service charges.¹⁵

APPENDIX I

Assuming an infinite horizon, equation (5) can be written as:

$$J_1[W(0)] = \int_0^{\infty} [\mu(d) - r] e^{-rt} \left\{ \int_{W=0}^{\infty} Wh(W,t) dW \right\} dt$$

14 Similar arrangements have been practiced between large institutional traders and financial institutions.

15 Should the "inflated cost", discussed in Footnote #10, causes concern here, simulation models can easily be run to find percentage rate of "inflation" between our closed-form solution and the solution obtained by numerical methods to determine a fairer service charge.

$$\begin{aligned}
&= \mu(d) \int_0^\infty e^{-rt} \left\{ \int_{W=0}^\infty Wh(W,t) dW \right\} dt \\
&\quad - \int_0^\infty re^{-rt} \left\{ \int_{W=0}^\infty Wh(w,t) dW \right\} dt.
\end{aligned}$$

Exchange the order of integration:

$$\begin{aligned}
J_1[W(0)] &= \mu(d) \int_{W=0}^\infty \int_0^\infty e^{-rt} Wh(W,t) dt dW \\
&\quad - \int_{W=0}^\infty \int_0^\infty re^{-rt} Wh(W,t) dt dW.
\end{aligned}$$

Integration by parts yields:

$$\begin{aligned}
J_1[W(0)] &= \mu(d) \left\{ \int_{W=0}^\infty \left[-\frac{(he^{-rt}W)}{r} \right]_0^\infty dW \right. \\
&\quad + \int_{W=0}^\infty \int_0^\infty \frac{1}{r} (e^{-rt}W \frac{\delta h}{\delta t}) dt dW \\
&\quad - \int_{W=0}^\infty \left[-he^{-rt}W \right]_0^\infty dW \\
&\quad + \int_{W=0}^\infty \int_0^\infty e^{-rt}W \frac{\delta h}{\delta t} dt dW \\
&= \frac{\mu(d)}{r} \left\{ W(0) + \int_0^\infty e^{-rt} \int_{W=0}^\infty W \frac{\delta h}{\delta t} dW dt \right. \\
&\quad \left. - \left\{ W(0) + \int_0^\infty e^{-rt} \int_{W=0}^\infty W \frac{\delta h}{\delta t} dW dt \right\} \right\}.
\end{aligned}$$

Assuming the dynamics for the working capital, as of equation (4), follow the Brownian motion process, with the two parameters $\alpha(F)$ and $\sigma(F)$ fixed, the term:

$$\{ W(0) + \int_0^\infty e^{-rt} \int_{W=0}^\infty W \frac{\delta h}{\delta t} dW dt \}, \quad (A1)$$

was shown by Frenkel and Jovanovic (1980) to be equivalent to:

$$\{ W(0) - \frac{\alpha(F)}{r} (1 - \Gamma) \},$$

where

Γ = the Laplace transform of $f[W(0),t]$, such that

$$\Gamma = \int_0^\infty e^{-rt} f[W(0),t] dt. \quad (A2)$$

Therefore,

$$J_1[W(0)] = \left[\frac{\mu(d)}{r} - 1 \right] \left[W(0) - \frac{\alpha(F)(1-\Gamma)}{r} \right]. \quad (A3)$$

Also, equation (6) can be shown to be (with infinite horizon):

$$J_2[W(0)] = \Gamma \{ c + G[W(0)] \}. \quad (A4)$$

Thus, the total cost G becomes:

$$G[W(0)] = \left[\frac{\mu(d)}{r} - 1 \right] \left[W(0) - \frac{\alpha(F)(1-\Gamma)}{r} \right] + \Gamma \{ c + G[W(0)] \}. \quad (A5)$$

Rearranging, equation (7) obtains.

APPENDIX II

Cox and Miller (1965) showed that:

$$\Gamma = \exp\left\{\frac{W(0)}{\sigma(F)^2} [\alpha(F) - (\alpha(F)^2 + 2r\sigma(F)^2)^{1/2}]\right\}.$$

Denote $R = \frac{1}{\sigma(F)^2} \{\alpha(F) - [\alpha(F)^2 + 2r\sigma(F)^2]^{1/2}\}$, then,

$\Gamma = \exp\{RW(0)\}$, and

$$\frac{\delta\Gamma}{\delta W(0)} = R\Gamma.$$

Also, define $\beta = 1/\Gamma$, so $\beta(0) = 1$ and

$$\frac{\delta\beta(t)}{\delta W(0)} \Big|_{t=0} = -R, \tag{A7}$$

$$\frac{\delta^2\beta(t)}{\delta W(0)^2} \Big|_{t=0} = R^2. \tag{A8}$$

Therefore, after rewriting equation (8) and dividing Γ through, it becomes:

$$\left[\frac{\mu(d)}{r} - 1\right](\beta - 1) + \left\{\left[\frac{\mu(d)}{r} - 1\right]W(0) + c\right\} = 0. \tag{A9}$$

Applying Taylor's Theorem, the term β can be expanded, ignoring terms of third and higher orders:

$$\beta = \beta(0) + W(0) \frac{\delta\beta(0)}{\delta W(0)} + \frac{W(0)^2}{2} \frac{\delta^2\beta(0)}{\delta W(0)^2} + \dots,$$

Substituting this expansion into equation (A9) and noting that $\beta(0) = 1$, it becomes:

$$\left[\frac{\mu(d)}{r} - 1 \right] \left[-RW(0) + \frac{R^2 W(0)}{2} \right] + \left[\frac{\mu(d)}{r} - 1 \right] RW(0) + cR = 0,$$

or

$$\left[\frac{\mu(d)}{r} - 1 \right] \frac{RW(0)}{2} + c = 0.$$

Therefore, equation (9) obtains.

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每日結算應備之流動性

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摘 要

期貨市場的每日結算制度目的在提供一低成本的信用保證。若價格走勢與投資者預期相反，此投資者須當日以現金補足價虧之數目。此制度使投資者免於對方拒不履約的風險。但它亦可能引發個人流利性不足的風險。本文應用現金管理原則及連續性時間隨機程序模型。求得一期初最適額之現金，以應付每日結算。若此帳戶枯竭，則在付出一固定成本後，可從一較高利率之定存帳戶移轉現金補充前一帳戶至期初最適額。依本模型操作可使避險者或投機者專注於其投資組合而不虞每日結算流動性風險之困擾。